



2019—2020 学年第一学期高二年级阶段性测评 数学试题参考答案及评分建议

一. 选择题:

1. D 2. B 3. A 4. D 5. C 6. B 7. C 8. A 9. C 10. D 11. C 12. B

二. 填空题:

13. $\sqrt{2}$ 14. $\frac{2\sqrt{2}\pi}{3}$ 15. $x^2 + y^2 = a^2 (a > 0)$ 16. $[\frac{3\sqrt{2}}{4}, \frac{\sqrt{5}}{2}]$

三. 解答题:

17 解: (1) 设 $C(x, y)$, 由题意得 $\begin{cases} -2 + x = 0, \\ -1 + y = 2, \end{cases} \therefore \begin{cases} x = 2, \\ y = 3, \end{cases} \therefore C(2, 3), \dots\dots\dots 2 \text{分}$

\therefore 直线 AC 的方程为 $x + 3y - 11 = 0$; $\dots\dots\dots 4 \text{分}$

(2) $\because A(-1, 4), C(2, 3), \therefore k_{AC} = -\frac{1}{3},$

$\therefore AC$ 边上的高所在直线的斜率 $k = 3,$ $\dots\dots\dots 6 \text{分}$

$\therefore AC$ 边上的高所在直线方程为 $3x - y + 5 = 0.$ $\dots\dots\dots 8 \text{分}$

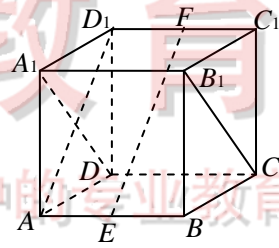
18. (1) 解: 连接 AD_1 , $\because ABCD - A_1B_1C_1D_1$ 是正方体, $\therefore AB \parallel C_1D_1, AB = C_1D_1,$

$\because E, F$ 分别是 AB, C_1D_1 的中点, $\therefore AE \parallel FD_1, AE = FD_1,$

$\therefore AEFD_1$ 是平行四边形, $\therefore EF \parallel AD_1,$ $\dots\dots\dots 3 \text{分}$

$\because EF \not\subset$ 平面 $ADD_1A_1, AD_1 \subset$ 平面 $ADD_1A_1,$

$\therefore EF \parallel$ 平面 $ADD_1A_1;$ $\dots\dots\dots 5 \text{分}$



(2) 由 (1) 得 $EF \parallel AD_1, \because ABCD - A_1B_1C_1D_1$ 是正方体,

$\therefore A_1B_1 \perp$ 平面 $ADD_1A_1, \therefore A_1B_1 \perp AD_1, \therefore A_1B_1 \perp EF,$

$\because ABCD - A_1B_1C_1D_1$ 是正方体, $\therefore ADD_1A_1$ 是正方形,

$\therefore A_1D \perp AD_1, \therefore A_1D \perp EF,$ $\dots\dots\dots 8 \text{分}$

$\because A_1D \subset$ 平面 $A_1B_1CD, A_1B_1 \subset$ 平面 $A_1B_1CD, A_1B_1 \cap A_1D = A_1,$

$\therefore EF \perp$ 平面 $A_1B_1CD.$ $\dots\dots\dots 10 \text{分}$

19 解: (1) $\because x^2 + y^2 = 1, \therefore C_1(0, 0), r_1 = 1,$ $\dots\dots\dots 2 \text{分}$

$\because x^2 + y^2 - 6x + m = 0, \therefore (x - 3)^2 + y^2 = 9 - m, \therefore C_2(3, 0), r_2 = \sqrt{9 - m}, \dots\dots\dots 4 \text{分}$

\because 圆 C_1 与圆 C_2 外切, $\therefore |C_1C_2| = r_1 + r_2, \therefore 3 = 1 + \sqrt{9 - m}, \therefore m = 5; \dots\dots\dots 5 \text{分}$

(2) 由 (1) 得 $m = 5,$ 圆 C_2 的方程为 $(x - 3)^2 + y^2 = 4, C_2(3, 0), r_2 = 2,$





由题意可得圆心 C_2 到直线 $x+2y+n=0$ 的距离 $d = \frac{|3+n|}{\sqrt{5}} = \sqrt{r_2^2 - 3} = 1$,8分

$\therefore n = -3 + \sqrt{5}$ 或 $n = -3 - \sqrt{5}$10分

20. (A) (1) 证明: $\because \triangle PAD$ 是正三角形, $AD = 2CD = 4$,

$\therefore PD = 4, CD = 2, \therefore PC^2 = PD^2 + CD^2 = 20, \therefore CD \perp PD$,2分

$\because AD \perp CD, \therefore CD \perp$ 平面 $PAD, \therefore CD \perp PA$;5分

(2) 设点 E 是 PD 的中点, 连接 AE, CE ,

$\because \triangle PAD$ 是正三角形, $\therefore AE \perp PD, AE = 2\sqrt{3}$,

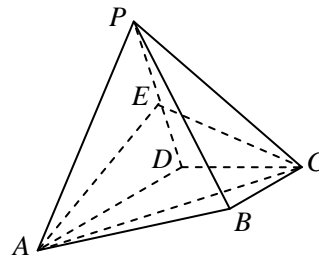
由(1)得 $CD \perp$ 平面 PAD, \therefore 平面 $PCD \perp$ 平面 PAD ,

$\therefore AE \perp$ 平面 PCD ,

$\therefore AC$ 与平面 PCD 所成角为 $\angle ACE$,8分

$\because AD \perp CD, \therefore AC = \sqrt{AD^2 + CD^2} = 2\sqrt{5}$,

$\therefore \sin \angle ACE = \frac{AE}{AC} = \frac{\sqrt{15}}{5}$10分



(B)图

21. (B) (1) 同 (A) (1);

(2) 设点 E 是 AD 的中点, 连接 PE, BE ,

$\because \triangle PAD$ 是正三角形, $\therefore PE \perp AD, PE = 2\sqrt{3}$,

$\because AD \parallel BC, \therefore BC \perp BE$,

$\because AD = 2BC = 2CD = 4, \therefore DE = BC = 2$,

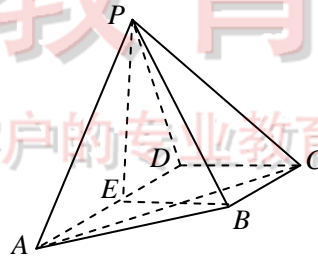
$\because AD \perp CD, AD \parallel BC, \therefore BCDE$ 是正方形,

$\therefore BC \perp BE, \therefore BC \perp$ 平面 $PBE, \therefore BC \perp PB$,

$\therefore \angle PBE$ 是二面角 $P-BC-A$ 的平面角,8分

由(1)得 $CD \perp$ 平面 $PAD, \therefore CD \perp PE, \therefore BE \perp PE$,

$\therefore \tan \angle PBE = \frac{PE}{BE} = \sqrt{3}, \therefore \angle PBE = 60^\circ$10分



(A)图

21 解: (A) (1) 设 $P(x, y), \therefore x^2 + y^2 = 4, \therefore O(0,0), r = 2$,

$\because |PA| = 2\sqrt{3}, \therefore |OP| = \sqrt{r^2 + |PA|^2} = 4$,2分

$\therefore \begin{cases} x^2 + y^2 = 16, \\ x - 2y - 8 = 0, \end{cases}$ 解得 $\begin{cases} x = 0, \\ y = -4, \end{cases}$ 或 $\begin{cases} x = \frac{16}{5}, \\ y = -\frac{12}{5}, \end{cases}$ 4分





$\therefore P(0, -4)$ 或 $P(\frac{16}{5}, -\frac{12}{5})$;5分

(2) 由题意可知当 $OP \perp l$ 时, $\angle APB$ 取最大值, 设此时 $P(x, y)$,

由 $\begin{cases} y = -2x, \\ x - 2y - 8 = 0 \end{cases}$ 得 $\begin{cases} x = \frac{8}{5}, \\ y = -\frac{16}{5}, \end{cases} \therefore P(\frac{8}{5}, -\frac{16}{5})$,8分

$\therefore \triangle APO$ 的外接圆方程为 $(x - \frac{4}{5})^2 + (y + \frac{8}{5})^2 = \frac{16}{5}$;10分

21. (1) 同 (A) (1);

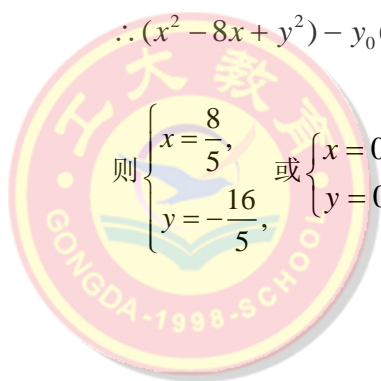
(2) 设 $P(x_0, y_0)$, 则 $M(\frac{x_0}{2}, \frac{y_0}{2})$,

$\therefore \triangle APO$ 的外接圆方程为 $x^2 - x_0x + y^2 - y_0y = 0$,7分

$\because x_0 - 2y_0 - 8 = 0, \therefore x_0 = 2y_0 + 8,$

$\therefore (x^2 - 8x + y^2) - y_0(2x + y) = 0$, 令 $\begin{cases} 2x + y = 0, \\ x^2 - 8x + y^2 = 0, \end{cases}$

则 $\begin{cases} x = \frac{8}{5}, \\ y = -\frac{16}{5}, \end{cases}$ 或 $\begin{cases} x = 0, \\ y = 0 \end{cases}$ (舍去), \therefore 圆 M 过定点 $(\frac{8}{5}, -\frac{16}{5})$10分



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